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Leakage-based precoding algorithms for multiple streams per terminal MU-MIMO systems

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ABSTRACT

Mobile networks rely extensively on multi-input multi-output (MIMO) communications to increase the data rate and improve signal quality. In multi-user MIMO (MU-MIMO) the signals are steered at the base station, forming multiple beams to the several user terminals (UTs). This is achieved by precoding the signals to be sent to the UTs. In leakage-based precoding, the precoding vectors are selected so that the signal to leakage plus noise ratio is maximized, where leakage is the amount of signal that is meant for a given UT but is received by the other UTs. This technique gives better results than techniques that completely eliminate the interference without regard to the signal or noise level (zero forcing solutions) like the block diagonal (BD) algorithm. However, current leakage-based algorithms are only optimal in the case of single-antenna UTs. In this paper, we propose simplified versions of these algorithms suitable for multiple streams per UTs, and compare its performance with existing solutions. Simulation results show an increase in performance. One of the versions does not require any Singular Value Decomposition (SVD) while the other does SVD of a much smaller matrix. Both still achieve better performance than the competition.

1. Introduction

The new generations of mobile networks (4G/5G) [1] use extensively multi-input multi-output (MIMO) systems [2], with several antennas at the base station (BS) and at the user terminals (UT) to improve the data rate, reduce radiated power and improve signal quality. It is now possible to serve several UTs simultaneously using what is called multi-user MIMO (MU-MIMO) as opposed to single-user MIMO (SU-MIMO). There are mainly two channels in MU-MIMO: the downlink channel, also called the broadcast channel (BC), and the uplink channel, also called the multiple access channel (MAC). This paper focus on the downlink channel. Orthogonal frequency division modulation (OFDM) [3] and perfect channel state information (CSI) is assumed through the paper.

In order to increase the data rate in telecommunication systems the signals are usually preprocessed before being sent to the channel by a linear or non-linear precoder, and processed by the receiver using a linear or non-linear decoder. In SU-MIMO systems, the conjunction of linear precoding and decoding can be used to transform the channel into a set of independent channels that can be treated almost independently to achieve capacity [4]. MU-MIMO systems are harder because joint demodulation cannot be implemented across UTs. In the case of MU-MIMO systems, the precoder and decoder may act as beamformers that direct the signal from the BS to the UT in the case of the precoder, and as spatial filters of the signals from the BS in the case of the decoder. In these systems, the use of non-linear interference canceling techniques are required to achieve capacity. Namely, it has been shown that the capacity region of the channel is achievable using dirty paper coding [5], but these techniques require further processing.

Through this paper, matrices are upper case boldface letters and vectors are bold face letters. $\mathbf{A}^T$, $\mathbf{A}^H$, trace($\mathbf{A}$), and $|\mathbf{A}|$ stand for the transpose, hermitian transpose, trace and determinant of $\mathbf{A}$, respectively. $\{\mathbf{A}\}_{i,j}$ stands for the entry at line i and column j of the matrix $\mathbf{A}$. The expectation is represented by E[·]. The notation diag($\mathbf{x}$) represents the diagonal matrix with $\mathbf{x}$ in its main diagonal.

2. State of the art

There has been much work done on MIMO systems for cellular communications. Following the results for MU-MIMO capacity there has been some work on papers with practical means to get it, namely, using vector perturbations [6,7], lattice precoding [8,9] and Tomlinson–Harashima precoding [10–12]. However, these methods

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are complex and may require to be used with some sort of linear coding and decoding. In this paper, we focus on linear methods.

The problem of linear joint precoder and decoder design for the downlink (DL) SU-MIMO has been addressed in [13,14]. The case of MU-MIMO is addressed in [15–18,10,19] etc. In [15] the problem is addressed using a zero-forcing approach, where the interference between the signals to different UTs is zeroed and the remaining degrees of freedom are used to transmit multiple streams to a single UT. This is called the block diagonal (BD) algorithm. This solution is not optimal because forcing the interference to be exactly zero is too strong, and reduces the available signal level. The following papers try to offer a solution to this problem.

The article [16] presents a solution but only for single-antenna UTs, where the transmit power is minimized while maintaining a given signal to interference plus noise ratio (SINR) in each UT. In [17] the authors use a conic optimizations approach: they allow for multiple-antennas UTs and assume predetermined decoders at the UTs.

In [18] the authors propose a leakage-based criterion. In this criterion, the signal to leakage plus noise ratio is maximized (SLNR), instead of SINR. The leakage is the signal that was meant to a given UT, but instead leaks to other UTs. This leads to a simple closed-form solution where the beamforming vectors can be calculated independently for each UT. They also present a multiple stream version, but with some approximations, namely: 1) using a matched filter decoder; 2) forcing total decoupling between streams of a single UT; 3) not optimizing for capacity but rather using the SLNR with the total power of the signal leakage and noise across all the UT antennas.

The article [20] presents a technique where the beamforming vectors are adapted using knowledge about the transmitted data, instead of independent of it. This results in cases where the interference can be taken as signal instead of noise, increasing the signal level by constructive interference. They derive low computational complexity implementation of the technique.

There are technique that simplify/improve the BD algorithm. In [21], first the minimum mean square error (MMSE) linear precoder is used to approximately invert the matrix formed by channel matrices of all the users; to get a set of almost orthogonal channels to each user. Second, the MMSE linear precoder is used again for each user but for a modified channel. The channel is modified by a matrix $\mathbf{T}$ obtained using a lattice reduction [22]. The matrix $\mathbf{T}$ is then used at the decoder, mimicking a SVD operation. This matrix is unimodular matrix, formed by integers, allowing a close to optimum implementation of the receiver. They use the QR decomposition to implement the inverses allowing a reduction of the computational complexity. In [19], the matrix inversion of [6] is approximated by using a truncated polynomial expansion. This is an approximate solution of an approximate solution.

In [23] they generalize leakage precoding techniques to filter bank multi-carrier modulation (FBMC) with offset quadrature amplitude modulation (OQAM). There are also applications of these techniques to visible light communications (VLC) [24].

Some other papers are more concerned with the case of very large number of antennas or massive MIMO [25,26].

This paper builds on the work of [18] by proposing simplified minimum leakage precoding for multiple streams per UT, but that still achieves better results according to simulations.

3. The channel model

The channel considered in the paper if formed by N base station antennas and M UTs. Each UT u has L_u antennas. Let,

The signal at the UT $\mathbf{y}_u$ is given by,

$$\mathbf{y}_u = \mathbf{H}_u \mathbf{x} + \mathbf{v}_u \tag{1}$$

where $\mathbf{H}_u$ are the channels matrices, $\mathbf{x}$ is the signal at the BS and $\mathbf{v}_u$ are the noise signals. The noise is taken as an independent identically distributed (i.i.d.) circular Gaussian distribution, with variance σ_v^2.

The signal at the BS is formed by the sum of the signals intended for each UT,

$$\mathbf{x}(n) = \sum_{u=1}^{M} \sqrt{q_u}\, \mathbf{W}_u \mathbf{s}_u(n), \tag{2}$$

where q_u controls the power of the signal sent to each UT. $\mathbf{W}_u$ is the beamforming matrix of UT u and $\mathbf{s}_u(n)$ is a size S_u vector of the streams intended for UT u. Note that equal power is sent to all UTs in all the studied algorithms. The autocorrelation matrices of the stream vector $\mathbf{s}_u(n)$ is given by $\mathbf{P}_u = \mathrm{diag}(\mathbf{p}_u)$ where $\mathbf{p}_u$ is the vector with the powers of each stream. At the UT, the received signal is processed by a decoding matrix $\mathbf{U}_u^H$, resulting in the signal $\hat{\mathbf{s}}_u(n)$ as in,

$$\hat{\mathbf{s}}_u(n) = \mathbf{U}_u^H \mathbf{y}_u(n). \tag{3}$$

The maximum rates (more accurately the spectral efficiency) that the studied algorithms can achieve are calculated using [27,4],

$$R_u = \log_2\left(\frac{|\mathbf{S}_u + \mathbf{N}_u|}{|\mathbf{N}_u|}\right) \tag{4}$$

with

$$\mathbf{S}_u = \mathbf{U}_u^H \mathbf{H}_u \mathbf{W}_u \mathbf{P}_u \mathbf{W}_u^H \mathbf{H}_u^H \mathbf{U}_u \tag{5}$$

$$\mathbf{N}_u = \mathbf{U}_u^H \left(\sum_{t=1, t\neq u}^{M} \mathbf{H}_u \mathbf{W}_t \mathbf{P}_u \mathbf{W}_t^H \mathbf{H}_u^H + \sigma_v^2 \mathbf{I} \right) \mathbf{U}_u \tag{6}$$

and the sum rate is given by $R = \sum_{u=1}^{M} R_u$. Where $\mathbf{S}_u$ is the signal autocorrelation matrix and $\mathbf{N}_u$ is the noise plus interference autocorrelation matrix of UT u.

4. The proposed algorithms

In [18] the authors present the optimal solution to the problem of selecting the precoder that maximizes the SLNR for single-antenna UTs. For the case of multiple-antenna UTs with multiple streams they present a suboptimal solution. In this paper, we propose a different approach to the multiple streams case, building on their solution for the single-antenna case, namely: to use a fixed known decoder in each UT and the single-antenna version of the leakage-based precoding algorithm to steer the beams in the direction defined by each stream.

Each UT can receive streams of data from multiple directions. In order to separate the streams of data from the multiple directions it can use multiple beamformers that are implemented by a decoder matrix. Given a decoder the requirement is to maximize the signal sent to a stream while minimizing the leakage to the other streams. This results in new low complexity and good performance algorithms. These algorithms are described in this section.

We propose two versions of the algorithm of different complexity. In the first version, the more complex one, we use singular value decomposition (SVD) to calculate the decoder matrix and a water-filling to choose the power for each stream. In the second version, the less complex one, the decoder matrix is simply the identity (no decoder and no SVD), and power is distributed evenly to all streams (no water-filling). We proceed to describe the first version.

Let $\mathbf{U}_u$, $\mathbf{\Sigma}_u$ and $\mathbf{V}_u$ be the SVD of $\mathbf{H}_u$, so that $\mathbf{H}_u = \mathbf{U}_u \mathbf{\Sigma}_u \mathbf{V}_u^H$ where $\mathbf{U}_u$ and $\mathbf{V}_u$ are unitary and $\mathbf{\Sigma}_u$ diagonal. The matrix $\mathbf{U}_u^H$ is the decoding matrix in our algorithm as defined in the previous section. This is the decoder matrix that would achieve capacity in the case of a SU-MIMO. $\mathbf{U}_u$ can also be calculated as the matrix whose columns are the orthonormal eigenvectors of $\mathbf{H}_u\mathbf{H}_u^H$. Then define the modified channel matrices as

$$\bar{\mathbf{H}}_u = \mathbf{U}_u^H \mathbf{H}_u. \tag{7}$$

Note that the number of streams is taken to be equal to the number of UT antennas, $L_u = S_u$. If one wishes to use fewer streams one can always select a few. Following the work of [18] the precoding matrices can be calculated as follows.

Let, $\mathbf{h}_{u,k}$ be the row vector that is the k-th row of the matrix $\bar{\mathbf{H}}_u$, and $\mathbf{w}_{u,k}$ be the k-th column of the matrix $\mathbf{W}_u$, where k goes from 1 to S_u. Then the SLNR at UT u, stream k is

$$\text{SLNR}_{u,k} = \frac{\mathbf{w}_{u,k}^H \mathbf{h}_{u,k}^H \mathbf{h}_{u,k} \mathbf{w}_{u,k}}{\mathbf{w}_{u,k}^H \mathbf{Q}_{u,k}\, \mathbf{w}_{u,k}} \tag{8}$$

with $\|\mathbf{w}_{u,k}\| = 1$ and,

$$\mathbf{Q}_{u,k} = \sum_{i=1, j=1, [i,j]\neq[u,k]}^{M, L_u} \mathbf{h}_{i,j}^H \mathbf{h}_{i,j} + \sigma_v^2/q_u \mathbf{I}\{\mathbf{U}_u^H \mathbf{U}_u\}_{k,k} =$$

$$\sum_{i=1}^{M} \bar{\mathbf{H}}_i^H \bar{\mathbf{H}}_i - \mathbf{h}_{u,k}^H \mathbf{h}_{u,k} + \sigma_v^2/q_u \mathbf{I}\{\mathbf{U}_u^H \mathbf{U}_u\}_{k,k}. \tag{9}$$

Note that in the first expression, the summation is for all values of i and j but for $[i, j] = [u, k]$ as represented and that in the present case $\mathbf{U}_u^H \mathbf{U}_u = \mathbf{I}$. Maximizing (8) and normalizing results in,

$$\mathbf{w}_{u,k} = \frac{\mathbf{Q}_{u,k}^{-1} \mathbf{h}_{u,k}^H}{\|\mathbf{Q}_{u,k}^{-1} \mathbf{h}_{u,k}^H\|}. \tag{10}$$

Also note that for the calculations the actual matrix inversion is not required, since solving a linear system is sufficient. Note that (10) does not use SVD nor matrix inversions, contrary to the solutions presented in [18]. This is only possible because $\mathbf{h}_{u,k}^H \mathbf{h}_{u,k}$ is a rank one matrix. The correctness of (10) can easily be verified by differentiation and substitution.

Then apply a water-filling algorithm to each UT separately. In order to do this, the channel gain to noise ratio, $\gamma_{u,k}$, for each stream k of each UT u needs to be calculated. This is given by,

$$\gamma_{u,k} = (\{\mathbf{S}_u\}_{k,k}/q_u)/\{\mathbf{N}_u\}_{k,k} \tag{11}$$

where

$$q_u = \frac{T_P}{M L_u} \tag{12}$$

and

$$\mathbf{P}_u = \text{diag}(\mathbf{p}_u) = q_u \mathbf{I} \tag{13}$$

and T_P is the total power sent to all the UTs, q_u is the average power per stream, $\bar{\mathbf{S}}_u/q_u$ is the channel gain matrix and $\mathbf{N}_u$ is the interference plus noise autocorrelation matrix from (5) and (6). Then, the new value of $\mathbf{p}_u = [p_{u,1} \dots p_{u,L_u}]^T$ is chosen by allocating power to each stream using,

$$p_{u,k} = \left(\mu - 1/\gamma_{u,k}\right)^+ \tag{14}$$

where the x^+ is defined as $\max(0, x)$, and μ is chosen so that $\sum_{k=1}^{L_u} p_{u,k} = T_P/M$.

Note that the actual value of the interference depends on the values chosen for the power sent in each stream, but we simply assume equal powers in the calculation. This still results in improved performance, as confirmed by simulations.

The second version of the algorithm can be deriving from the first version by simply making $\mathbf{U}^H = \mathbf{I}$ and making $P_{u,k} = q_u$. Note that there is one third possibility, that will not be studied in this paper, which is to use a matched filter for the decoder, namely $\mathbf{U}^H = \mathbf{H}\mathbf{H}^H$.

Regarding the computational complexity, the first version of the proposed algorithm only calculates the eigenvectors of M small $L_u \times L_u$ matrices, while [18] needs to calculate the generalized eigenvectors of M large $N \times N$ matrices. The BD algorithm [15] needs to calculate the null space of a $L_u \times N$, which can be done using SVD, and then a SVD of $S_u \times S_u$ matrices. The second version of the proposed algorithm does not require any SVD decomposition.

5. Other algorithms

In this section a short description of the algorithms used in the simulation section is presented.

5.1. Sadek algorithm

The algorithm in [18] was slightly modified to follow the nomenclature of the paper. The transmit power T_p is adjusted by $\mathbf{P}_u = q_u \mathbf{I}$ resulting in $T_P = \sum_{u=1}^{M} \text{trace}\{\mathbf{W}_u^H \mathbf{P}_u \mathbf{W}_u\} = q_u M L_u$ where $q_u L_u$, the power per UT, is actually independent of u and all stream signals, $\mathbf{s}_u(n)$, are taken to have unit power.

Let $\mathbf{T}_u$ be the matrix with columns formed by generalized eigenvectors of the matrices $\mathbf{A}_u = \mathbf{H}_u^H \mathbf{H}_u$ and $\mathbf{B}_u = \sigma_i^2 \mathbf{I} + q_u \tilde{\mathbf{H}}_u^H \tilde{\mathbf{H}}_u$. The matrix $\tilde{\mathbf{H}}_u$ is defined by $\tilde{\mathbf{H}}_u = [\mathbf{H}_1^T \dots \mathbf{H}_{u-1}^T \mathbf{H}_{u+1}^T \dots \mathbf{H}_M^T]^T$. Let $\mathbf{D}_u$ be a diagonal matrix with the generalized eigenvalues of the same matrices. One has that $\mathbf{A}_u \mathbf{T}_u = \mathbf{B}_u \mathbf{T}_u \mathbf{D}_u$. Assume that the eigenvalues are stored in descending order, i.e. $\{\mathbf{D}_u\}_{i,i} < \{\mathbf{D}_u\}_{j,j}$ for $i > j$. Then $\mathbf{W}_u$ is given by first L columns of $\mathbf{T}_u$ and is then normalized to set the transmitted power to each UE equal to T_P/M. In this paper it is assumed that the decoder matrices are set to $\mathbf{U}_u^H = \mathbf{W}_u^H \mathbf{H}_u^H$.

5.2. Block diagonalization algorithm

In this section the Block Diagonalization (BD) algorithm is discussed [15].

In order to zero the interference on other UTs from the signal transmitted by the kth UT, one must have

$$\tilde{\mathbf{H}}_u \mathbf{W}_u = \mathbf{0} \tag{15}$$

with $\tilde{\mathbf{H}}_u$ as,

$$\tilde{\mathbf{H}}_u = [\mathbf{H}_0^T, \dots, \mathbf{H}_{u-1}^T, \mathbf{H}_{u+1}^T \dots \mathbf{H}_M^T]^T \tag{16}$$

so the L_u columns of $\mathbf{W}_u$ should lie on the null space of $\tilde{\mathbf{H}}_u$. In this paper this is implemented by selecting $\mathbf{W}_u$ from the L_u lowest singular value vectors of $\tilde{\mathbf{H}}_u$. This will work even in cases where these singular values are not exactly zero, but close to zero, minimizing the interference between channels. Taking the interference as zero, the channels of the MU-MIMO can be decoupled, resulting in

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \mathbf{v}_k \tag{17}$$

Taking $\mathbf{H}_k \mathbf{W}_k$ as the channel matrices of a set of independent SU-MIMO channels the full capacity of these channels can be achieved using SVD.

Table 1
Computation complexity of several matrix operations.

	flops
LU factorization	$2/3n^3$
back substitution	n^2
forward substitution	n^2
solving a liner system (SLS)	$2/3n^3 + 2n^2$
generalized eigenvectors (GEV)	$14n^3$
eigenvectors (EV)	$9n^3$
economy R-SVD (ESVD)	$6mn^2 + 20n^3$
Golub–Reinsch SVD (SVD)	$21n^3$

Table 2
Computational complexity of the BD algorithm.

	flops
for u = 1 to M	
$\tilde{\mathbf{H}}_u = [\mathbf{H}_0^T, \ldots, \mathbf{H}_{u-1}^T, \mathbf{H}_{u+1}^T \ldots \mathbf{H}_M^T]^T$	
$\mathbf{U}_u^0 \boldsymbol{\Sigma}_u^0 \mathbf{V}_u^{0H} = \tilde{\mathbf{H}}_u$ % ESVD	$6L(M-1)N^2 + 20N^3$
$\mathbf{W} = \mathbf{V}^0(:, end - L + 1 : end)$	
$\mathbf{U}_u^1 \boldsymbol{\Sigma}_u^1 \mathbf{V}_u^{1H} = \mathbf{H}_u \mathbf{W}$ % SVD	$21L^3$
$\mathbf{U}_u = \mathbf{U}_u^1$	
$\boldsymbol{\Sigma}_u = \boldsymbol{\Sigma}_u^1$	
$\mathbf{V}_u = \mathbf{W}\mathbf{V}_u^1$	$2NL^2$
water-filling calculations:	
$\gamma_{u,k} = \{\boldsymbol{\Sigma}_u^1\}_{k,k}/\sigma_v^2$	L
water-filling sorting and select μ	$O(L)$
end for	

total flops: $M(21L^3 + LM + 2L^2N + 6L(M-1)N^2 + 20N^3)$

Table 3
Computational complexity of Sadek's algorithm.

	flops
$\mathbf{Q} = \sum_{u=1}^M \mathbf{H}_u^H \mathbf{H}_u$	$2MLN^2$
for u = 1 to M	
$\mathbf{A}_u = \mathbf{H}_u^H \mathbf{H}_u$	$2LN^2$
$\mathbf{B}_u = q_u(\mathbf{Q} - \mathbf{A}_u) + \sigma_i^2 \mathbf{I}$	$2N^2 + N$
$\mathbf{A}_u \mathbf{T}_u = \mathbf{B}_u \mathbf{T}_u \mathbf{D}_u$ % GEV	$14N^3$
sort $\mathbf{T}_u$ and $\mathbf{D}_u$	$O(N\log_2(N))$
$\mathbf{W}_u = \mathbf{T}_u(:, 1:L)$	
$\mathbf{W}_u = \sqrt{L/\text{trace}(\mathbf{W}_u^H \mathbf{W}_u)}\mathbf{W}_u$	$3N^2 + N + 2$
$\mathbf{U}_u = \mathbf{W}_u^H \mathbf{H}_u^H$	$2L^2N$
end for	

total flops: $M(2 + 2N + 2L^2N + 5N^2 + 4LN^2 + 14N^3)$

6. Computational complexity

In this section the computational complexity of the algorithms discussed in the paper is studied.

To do this the number of floating points operations (flops) of each algorithm is studied. Additions, multiplications, divisions and square roots of complex numbers are counted as one flop. Note that using flops to measure computational complexity is still a crude approximation. The flop count of matrix operations used in the algorithm are presented in Table 1 and were taken from [28]. Note that [28] removes low order terms from the flops count. Table 1 also defines some acronyms that are used down in the text.

Next, in Table 2, 3 and 4 are presented in complexities of the BD [15], Sadek's [18] proposed 1 and proposed 2 algorithms, along with short descriptions of them. Table 4 only shows the description of proposed 1 and not proposed 2 because they are similar. The complexity of proposed 2 algorithm can be obtained by removing some of the calculation of the proposed 1 algorithm, for instance the EV calculations, and is shown in the end of the table. In the tables the symbol % is used to start comments. Comments indicate what is the operation in Table 1 that is used in the preceding calculation.

Table 4
Computational complexity of proposed algorithm.

	flops
for u = 1 to M	
$\mathbf{A}_u = \mathbf{H}_u \mathbf{H}_u^H$	$2L^2N$
$\mathbf{U}_u \boldsymbol{\Sigma}_u \mathbf{U}_u^H = \mathbf{A}_u$ % EV	$9L^3$
$\bar{\mathbf{H}}_u = \mathbf{U}_u^H \mathbf{H}_u$	$2L^2N$
end for	
$\mathbf{Q} = \sum_{u=1}^M \bar{\mathbf{H}}_u^H \bar{\mathbf{H}}_u$	$2MN^2L$
for u = 1 to M	
for k = 1 to L	
$\mathbf{Q}_{u,k} = \mathbf{Q} - \mathbf{h}_{u,k}^H \mathbf{h}_{u,k} + q_u \mathbf{I}$	$2N^2 + N$
$\mathbf{w}_{u,k}^0 = \mathbf{Q}_{u,k}^{-1} \mathbf{h}_{u,k}^H$ % SLS	$2/3N^3 + 2N^2$
$\mathbf{w}_{u,k} = \mathbf{w}_{u,k}^0 / \lVert \mathbf{w}_{u,k}^0 \rVert$	$3N$
end for	
end for	
water-filling calculations:	
for u = 1 to M	
$\mathbf{s}_u / q_u = \text{diag}(\bar{\mathbf{H}}_u \mathbf{W}_u \mathbf{W}_u^H \bar{\mathbf{H}}_u^H)$	$2L^2N + 2L^2$
$\mathbf{n}_u = \text{diag}\left(\sum_{t=1, t\neq u}^M \bar{\mathbf{H}}_u \mathbf{W}_t q_t \mathbf{W}_t^H \bar{\mathbf{H}}_u^H + \sigma_v^2 \mathbf{I}\right)$	$(M-1)(2L^2N + 3L^2) + L$
$\gamma_{u,k} = (\{\mathbf{s}_u\}_k / q_u)/\{\mathbf{n}_u\}_k$	L
water-filling	$O(L)$
end for	

total flops: $LM(2 - L + 9L^2 + 3LM + 4N + 4LN + 2LMN + 6N^2 + 2/3N^3) + MO(L)$

proposed 2
total flops: $LM(4N + 6N^2 + 2/3N^3)$

In the calculation of $\mathbf{s}_u$ and $\mathbf{n}_u$ of the proposed algorithm some considerations are in order. For $\mathbf{s}_u$ the expression is obtained by the following order of the operations:

$$\mathbf{A} = \mathbf{H}_u \mathbf{W}_u$$
$$\mathbf{s}_u = \text{diag}(\mathbf{A}\mathbf{A}^H).$$

The second expression is calculated using,

$$\{\mathbf{s}_u\}_k = \sum_{i=0}^{L} \{\mathbf{A}_u\}_{k,i}^2.$$

For $\mathbf{n}_u$ similar calculations are performed. The implementation of the water-filling procedure implies first the calculation of the channel gain to noise ratio $\gamma_{u,k}$ where instead of the noise the noise plus interference of each stream is used. Next, μ is selected by an iterative process [29] that uses $O(L)$ flops.

In Table 5 are presented numerical values for the computational complexity of the algorithms. Three sets of values were used. Note that the $O(L)$ term is not considered since it is a low value. As can be seen all algorithms have similar complexity but the proposed ones have the lowest values. This difference is bigger for lower values of L.[1]

7. The channel simulation model

The simulations in this paper use a largely simplified IMT-Advanced channel model [30]. A uniform linear array (ULA) is

[1] The complexity of the proposed algorithm can still be significantly reduced by using the Woodbury matrix identity to calculate $\mathbf{Q}_{u,k}^{-1}\mathbf{h}_{u,k}^H$ from $\mathbf{Q}^{-1}\mathbf{h}_{u,k}^H$.

Table 5
Numerical values for the computational complexity in flops.

	$N=32$, $M=10$, $L=2$	$N=16$, $M=4$, $L=4$	$N=32$, $M=10$, $L=4$
BD	7.66×10^6	409×10^3	8.79×10^6
Sadek's	4.73×10^6	253×10^3	4.81×10^6
Proposed 1	0.595×10^6	84.6×10^3	1.26×10^6
Proposed 2	0.562×10^6	69.3×10^3	1.12×10^6

assumed, clusters are not considered, neither are polarization and the antenna elements field patterns. The UT speed is taken as zero. Accordingly, the channel impulse response for each UT, u, of the model is given by,

$$\{\mathbf{H}_u(\tau)\}_{k,s} = H_{k,s}(\tau), \tag{18}$$

where the dependence on u was dropped to simplify the notation, k, is the UT antenna and s is the base station antenna. One further has,

$$H_{k,s}(\tau) = \sum_{m=1}^{L_u} \sqrt{P_m}\exp(2\pi isd/\lambda\sin(\phi_m)) \exp(2\pi ikd/\lambda\sin(\varphi_m))\delta(\tau-\tau_m) \tag{19}$$

where P_m, τ_m, ϕ_m and φ_m are the power, delay, angle of departure (AoD) and angle of arrival (AoA) of the ray m. The variables d, and λ are, respectively, the antenna spacing and the carrier wavelength. The Dirac delta function is $\delta(t)$. The number of rays was taken to be equal to the number of UT antennas.

All rays were taken to have equal power, $P_m = P$, unless otherwise stated. The delays of the rays, τ_m, were evenly spaced around the delay spread, DS. The angle of departure and angle of arrival are evenly spaced around a random direction that is a slow function of the UT position. Namely, a new parameter was introduced, reflection distance, RD, that is related to the change in the AoD or AoA per unit change in the UT position. If there is a reflection at a distance of RD, then when a UT moves by δ this causes an angle change of $2\pi\delta/\mathrm{RD}$ rad in the direction of the ray. The values taken by each of the three parameters as a function of the ray index m are grouped into three vectors. Finally, these vectors are shuffled in order to implement a random coupling between the parameters.

Note that MU-MIMO systems are highly sensitive to relations between the channels matrices, $\mathbf{H}_u$, so the model produces similar matrices for closely spaced UTs. This is not the case of the model in [30] that only correlates the large-scale parameters.

8. Simulation results

Figs. 1 and 2 serve to better illustrate the concept of leakage precoding and its advantages to zero forcing solution as the BD algorithm. The figures show a simulation of a system with 16 base station antennas and 2 UTs with 2 antennas. Fig. 1 shows the radiations pattern of two streams to UT 1 using the BD algorithm. In each of the streams the forcing of a radiation zero in the direction of UT 2 close to the direction of UT 1 limits the amplitude of the transmitted signal. Fig. 2 shows the same chart using the proposed algorithm. It can be seen that the resulting transmitted signal amplitudes are much higher.

The remaining simulation are based on a system with $N=16$ base station antennas, $L=4$ UT antennas, $M=4$ UTs, angular spread of arrival of ASA $=60°$, angular spread of departure of ASD $=60°$, delay spread of 1 µs, bandwidth of $B=20$ MHz, carrier frequency of $f_c=3$ GHz, half wavelength antenna spacing of $d=\lambda/2$, reflecting distance of RD $=200$ m and UTs at random positions inside a square with center at $x=300$ m, $y=0$ m and side

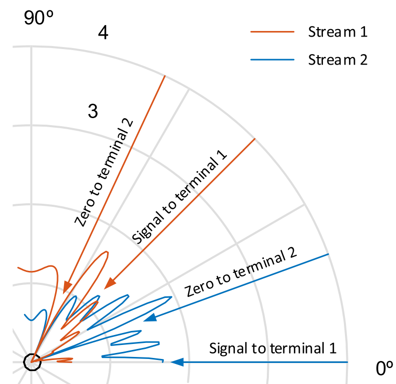

Fig. 1. Radiation patterns of two streams sent to UT 1 using the BD algorithm.

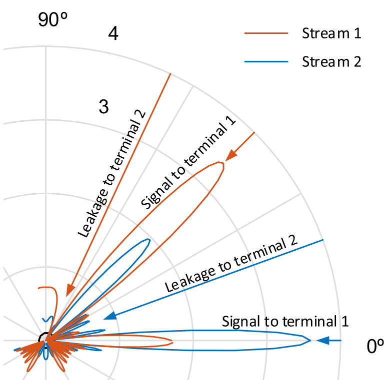

Fig. 2. Radiation patterns of two streams sent to UT 1 using the proposed algorithm.

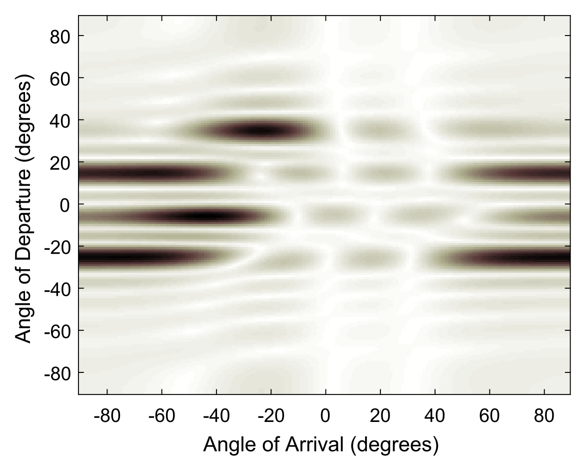

Fig. 3. Channel gain when varying the angle do departure and angle of arrival of the downlink channel. Dark areas mean higher gain light areas mean lower gain.

length of 100 m. Then some of the parameters are varied in each simulation as described below.

In order to visualize the channel from the base station to one of the UTs that results from such a system, the variation of the channel gain when both the transmitter and receiver are configured to simple delay and sum beamformers with varying angles is plotted in Fig. 3. It can be seen that there are four rays (the darker areas) that connect the base station to the UT with different angles of departure and angles of arrival. We use the concept of rays as defined in the IMT-Advanced channel model. Note that the figure repeats every 180 degrees.

The Figs. 4 to 6 show plots of the sum rate versus the UT distance for different system configurations. The UTs are all evenly spaced around a circumference with center at $x=300$ m and

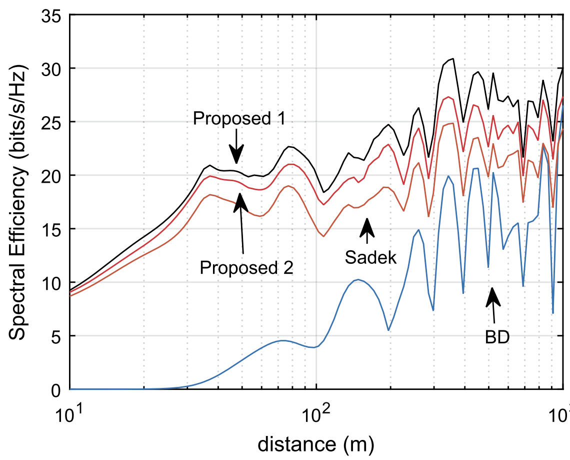

Fig. 4. Plot of the capacity versus the UTs distance for the proposed, Sadek and BD algorithms using a system with 16 base station antennas and 4 UTs with 4 antennas each.

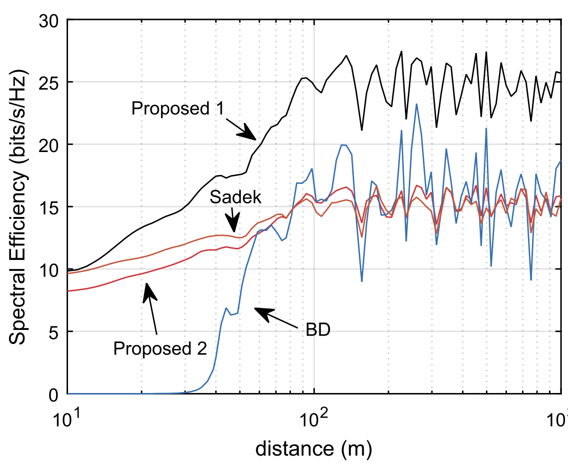

Fig. 5. Plot of the capacity versus the UTs distance for the proposed, Sadek and BD algorithms using a system with 16 base station antennas and 16 UTs with 4 antennas each.

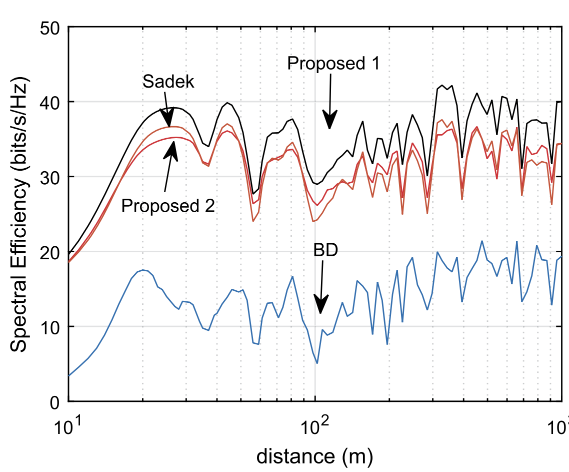

Fig. 6. Plot of the capacity versus the UTs distance for the proposed, Sadek and BD algorithms using a system with 32 base station antennas and 4 UTs with 4 antennas each.

$y = 0$ m. The rate typically grows with distance until it reaches a threshold. From this point on the UTs are easily separable and further increase in the distance no longer implies an increase in capacity. The proposed algorithms show much better performance than the BD [15] algorithm and are better than Sadek's [18] algorithm in any case. The BD algorithm performs very badly for close spaced UTs.

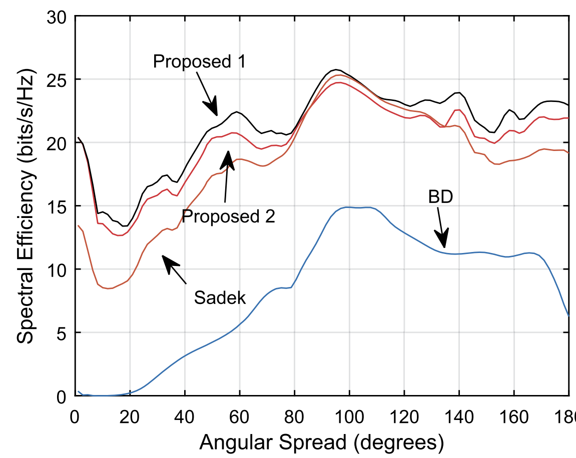

Fig. 7. Plot of the capacity versus the angular spread for the proposed, Sadek and BD algorithms.

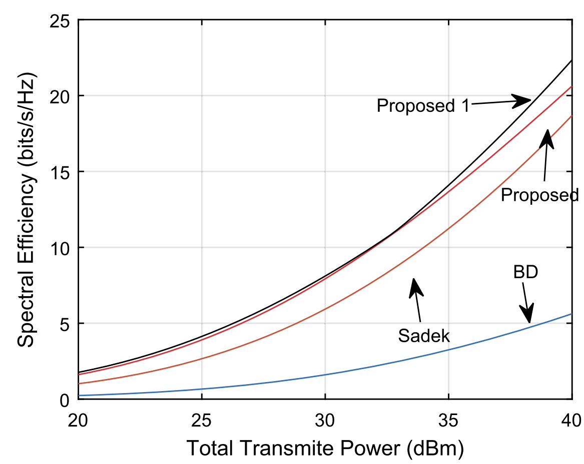

Fig. 8. Plot of the capacity versus the total transmit power, for the proposed Sadek and the BD algorithms.

Fig. 7 plots the variation of the sum rate with angular spread at the BS and UT. The proposed algorithms usually outperform the other two algorithms.

Finally in Fig. 8 compares the algorithm for changing BS transmit power. The proposed algorithms achieves gains of more than 2 dB to the Sadek's algorithm and more than 12 dB to the BD algorithm.

The proposed 1 and 2 algorithm perform similar in all Figs. but in Fig. 5 where the first version is better.

9. Conclusions

This paper proposes new algorithms for choosing the precoding matrices of MU-MIMO with multiple-antenna UTs based on the leakage criterion. The algorithms achieve better performance in simulations while having lower requirements on SVD. Namely, the second version of the algorithm does not require any complex SVD calculations contrary to alternative algorithms. The first version does require SVD but only in a small matrix. Compared to zero forcing solutions, the leakage-based criteria results in higher performance. It approximately solves the compromise between having high signals and low interference. Compared to other leakage based algorithms the proposed algorithm is suitable to multi-stream communication without forcing zero interference between streams to the same terminal, also resulting in better performance.

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